Lessons Learned from Our Research in Ontario Classrooms

Shelley Yearley

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Earlier work includes coaching within TLDSB, planning and co-facilitating Leadership PLMLC series and leading TLDSB's GAINS Literacy Question Structure Response for Mathematics project. She has previously served as the Steering Team Lead of the 2008-09 Coaching for Math GAINS initiative, co-facilitator of multiple Adobe Connect Book Studies, and lead for LMS and TIPS4RM resource development. Shelley is a member of the Math CAMPPP organizing team.

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Dr. Catherine D. Bruce

Cathy, cathybruce@trentu.ca, is an Associate Professor at Trent University, in Peterborough, Ontario, Canada where she teaches mathematics methods courses at the School of Education and Professional Learning. Cathy collaborates with teachers and researchers to engage in, and assess, professional learning models focused on mathematics and technology use, and she researches the effects of these activities on teachers and students. Recent speaking engagements include the Institute of Education at the University of London, AERO, MISA, and the Ontario Education Research Symposium. She is currently working on a federal research grant project (SSHRC) focused on mathematics for young children and the use of video for analysis of teacher and student learning. Her research can be accessed at www.tmerc.ca.

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3 out of 2 people have trouble with fractions….

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Road Map for Plenary 6

We will:

- outline the research (international & provincial)
- engage in a fractions matching task
- examine student thinking:
  - What do they REALLY understand?
  - Which representations do they rely on and why?
- think about number lines
- view a digital paper on fractions learning

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Why Fractions?

- Students have intuitive and early understandings of ½ (Gould, 2006), 100%, 50% (Moss & Case, 1999)
- Teachers and researchers have typically described fractions learning as a challenging area of the mathematics curriculum (e.g., Gould, Outhred, & Mitchelmore, 2006; Hiebert 1988; NAEP, 2005).
- The understanding of part/whole relationships & part/part relationships, procedural complexity, and challenging notation, have all been connected to why fractions are considered an area of such difficulty. (Bruce & Ross, 2009)

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Why Fractions?

- Students also seem to have difficulty ***retaining*** fractions concepts (Groff, 1996).
- Adults continue to struggle with fractions concepts (Lipkus, Samsa, & Rimer, 2001; Reyna & Brainerd, 2007) even when fractions are important to daily work related tasks.
- "Pediatricians, nurses, and pharmacists…were tested for errors resulting from the calculation of drug doses for neonatal intensive care infants… Of the calculation errors identified, 38.5% of pediatricians' errors, 56% of nurses' errors, and 1% of pharmacists' errors would have resulted in administration of 10 times the prescribed dose." (Grillo, Latif, & Stolte, 2001, p.168).

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We grew interested in…

- What types of representations of fractions are students relying on?
- And which representations are most effective in which contexts?

We used Collaborative Action Research to learn more.

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Collaborative Action Research

CLASSROOM

Collaboration

Throughout collaborative action research activity, teachers frequently meet to set goals, and to plan and engage in related interventions, data collection, data analysis and report writing. The involvement of researchers and knowledgeable others can range from full membership in the team to a supporting role (e.g., providing resources, assisting with data collection and analysis strategies).

Evaluating Effects

- analyzing data
- report writing
- sharing reports

Implementing Plan of Action

- enacting the plan in the classroom
- observing, co-teaching, supporting team members
- collecting evidence and reflecting

Goal Setting and Planning

- planning interventions that will improve teaching and learning
- consulting current research and accessing human/print resources
- setting timelines

Identifying the Problem

- articulating the teacher/learning issue for investigation
- gathering baseline data
- developing research question(s)

Asking New Questions

The findings of collaborative action research often reveal other problems and lead to new questions that provide a springboard for a second cycle of action research in new or refined areas of study.

Data Collection and Analysis

Data Collection and Analysis

AS A STARTING POINT

- Literature review
- Diagnostic student assessment (pre)
- Preliminary exploratory lessons (with video for further analysis)

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Data Collection and Analysis

THROUGHOUT THE PROCESS

- Gathered and analysed student work samples
- Documented all team meetings with field notes and video (transcripts and analysis of video excerpts)
- Co-planned and co-taught exploratory lessons (with video for further analysis after debriefs)
- Cross-group sharing of artifacts

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Data Collection and Analysis

TOWARD THE END OF THE PROCESS

- Gathered and analysed student work samples
- Focus group interviews with team members
- 30 extended task-based student interviews
- Post assessments

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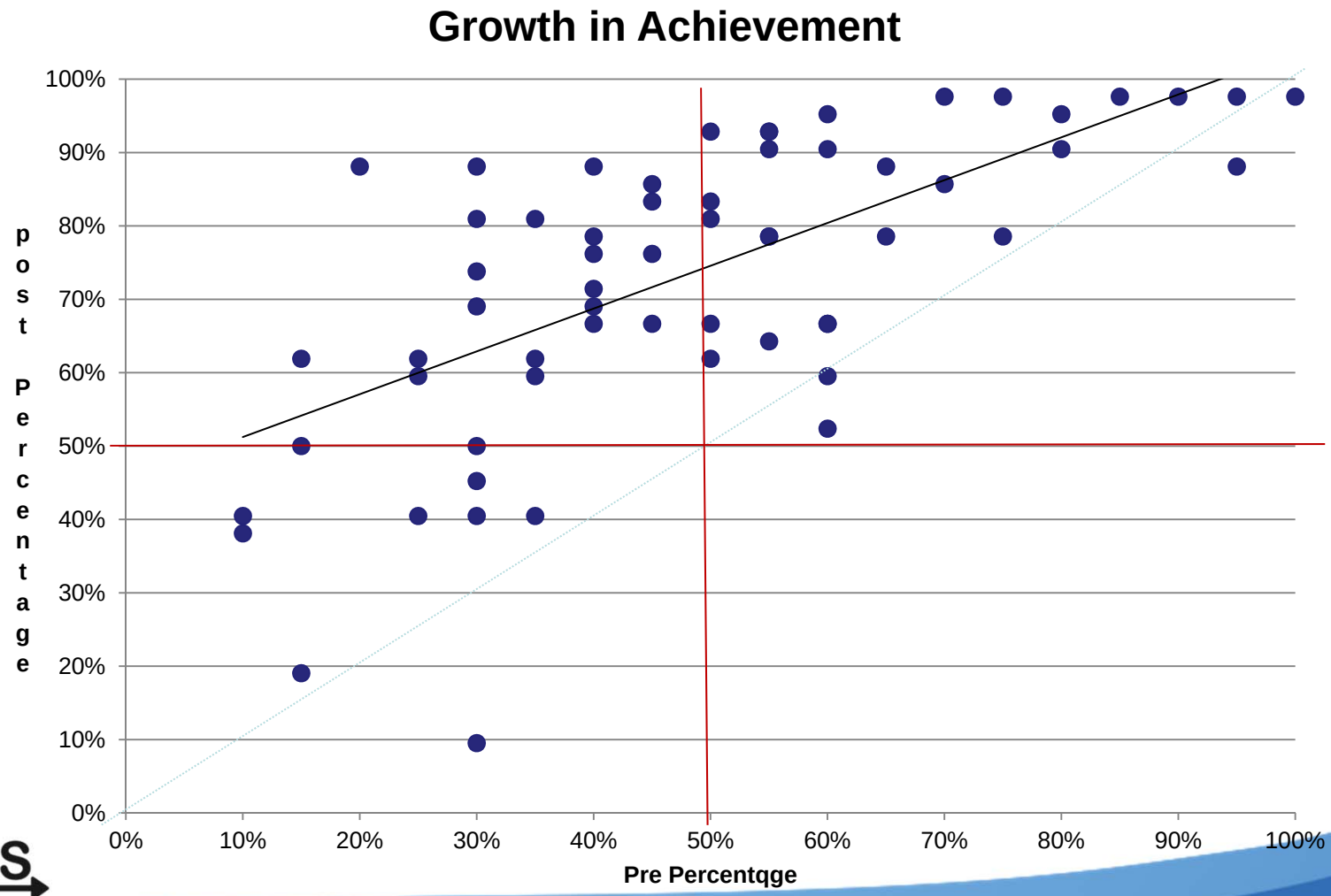
Student Results



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Envelope: “Matching Game”

There are 5 triads

MATCH 3 situation cards to symbolic cards and pictorial representation cards

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Match a situation to one of these…

- Linear relationship
- Part-whole relationship
- Part-part relationship
- Quotient relationship
- Operator relationship

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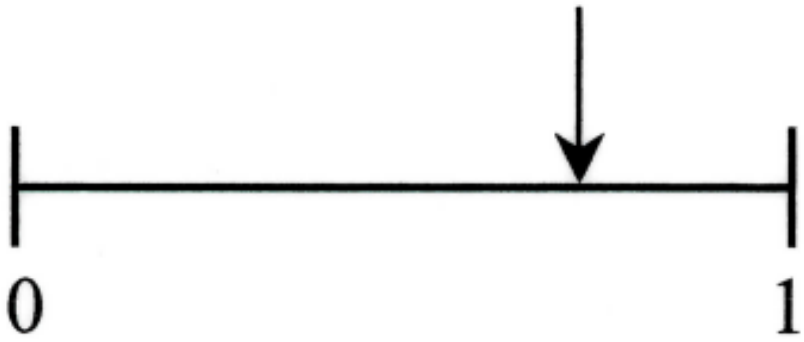
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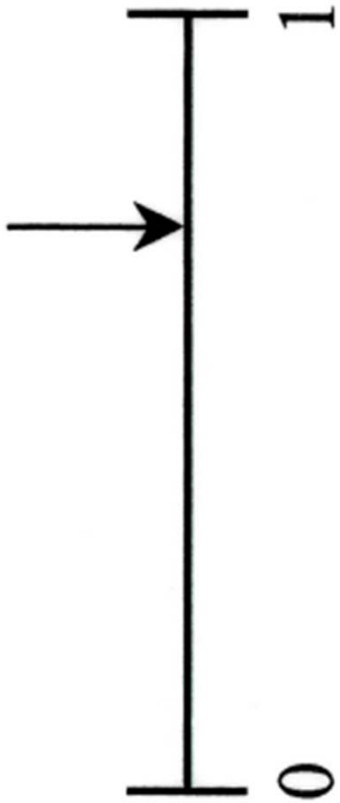
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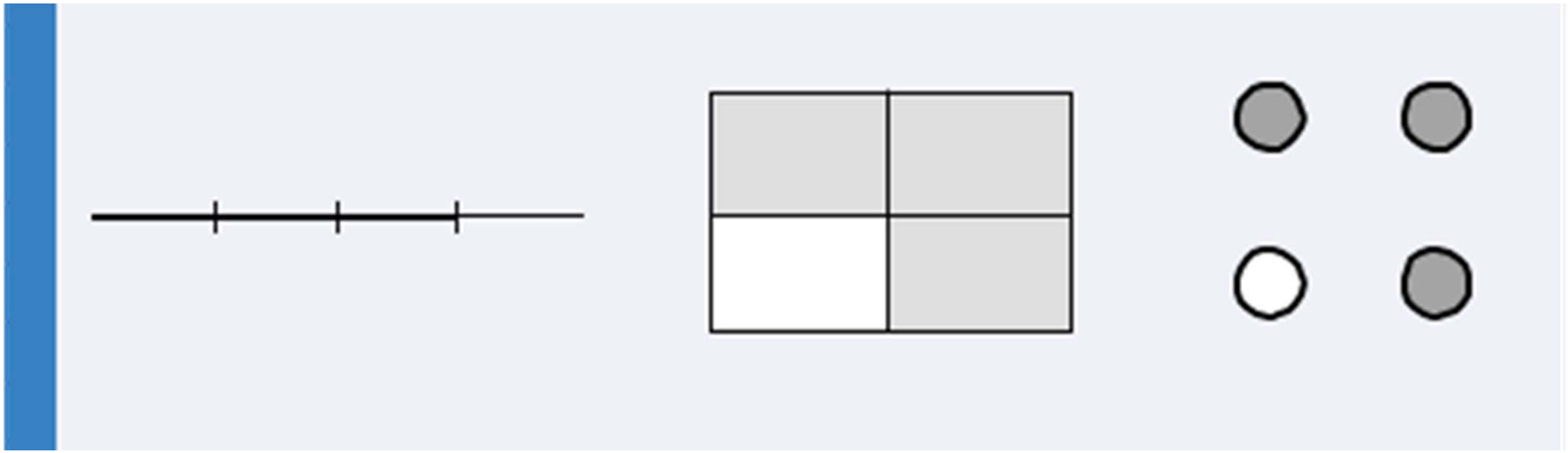
In our study…

We focused particularly on these three



Tad Watanabe, 2002

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Early Findings

- Students had a fragile and sometimes conflicting understanding of fraction concepts when we let them talk and explore without immediate correction
- Probing student thinking uncovered some misconceptions, even when their written work appeared correct
- ‘Simple’ tasks required complex mathematical thinking and proving

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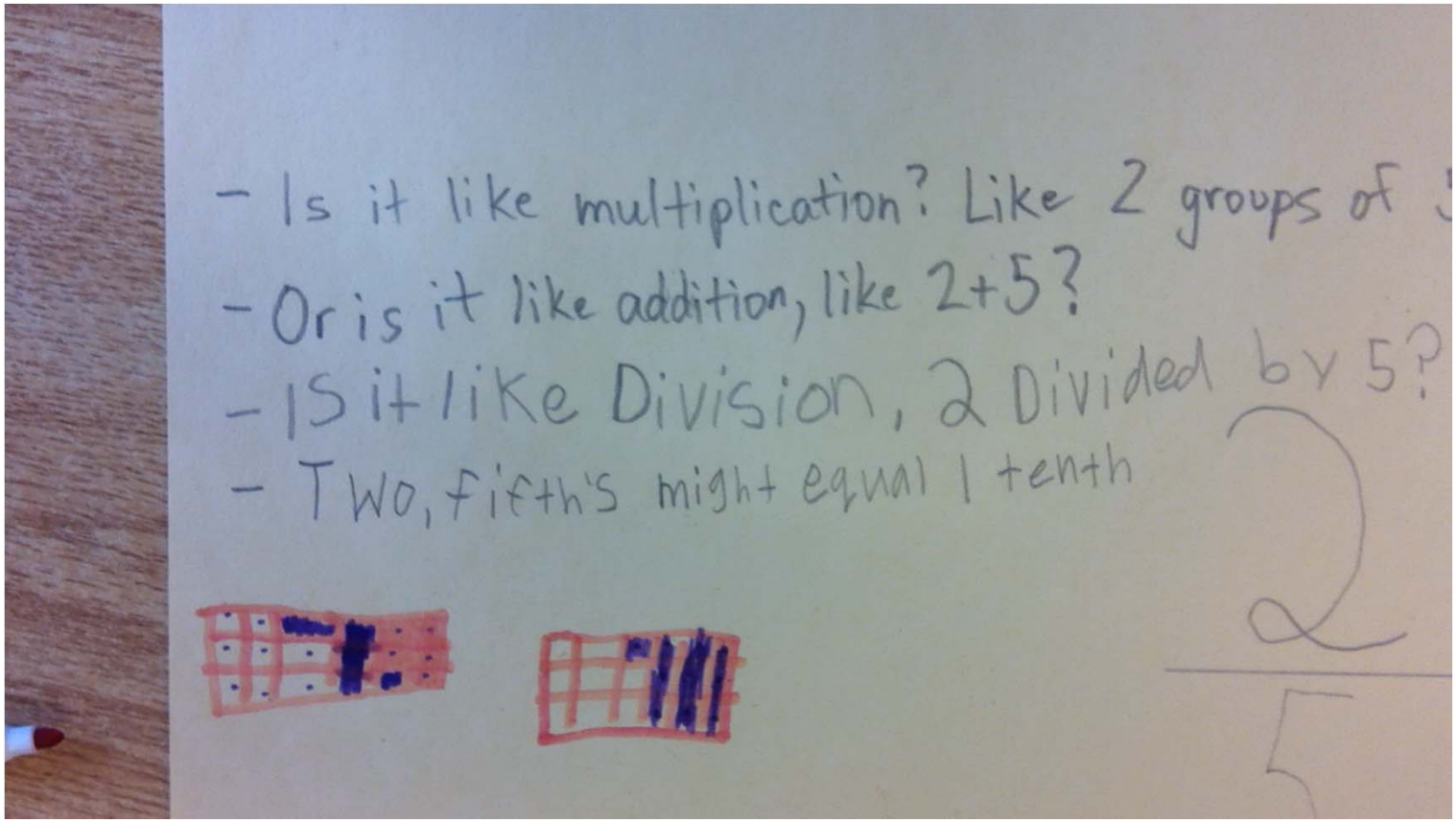
Represent 2/5 or 4/10



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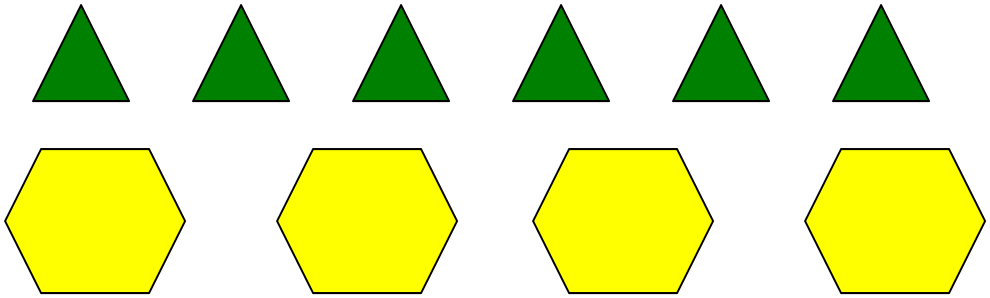
www.tmerc.ca/video-studies/


Ratio thinking?
St 2: So we know that we've covered those ones in.

Remember…

How can you name this?



6 green:4 yellow

Part-Part (set)

6 green:10 shapes

Part-Whole (set)

One fifth of the total area is green

Part-Whole (area)

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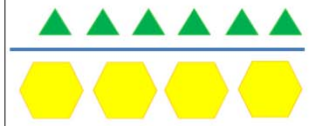
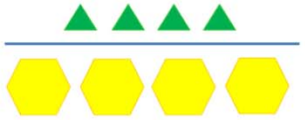
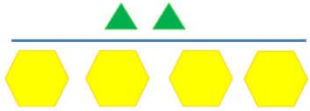
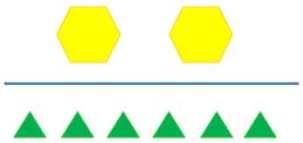
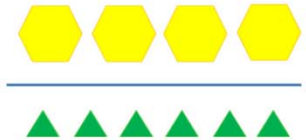
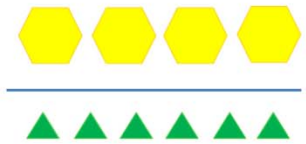
Holding Conflicting Meanings Simultaneously

Use the pattern blocks to create as many fractions as possible.

Pictures	Numbers	Words
	$\frac{6}{4}$	A fraction has a numerator and a denominator. The numerator tells you how many pieces are shaded. The denominator tells you how many pieces the whole was cut into.
	$\frac{4}{4}$	
	$\frac{2}{4}$	
	$\frac{2}{6}$	
	$\frac{4}{6}$	
	$\frac{4}{6}$	

What do the students understand? Are some understandings fragile?

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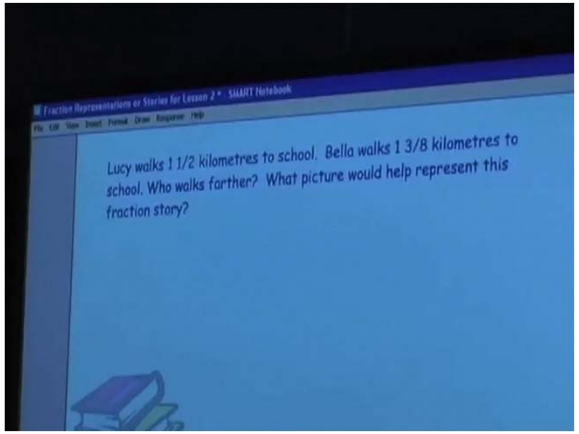
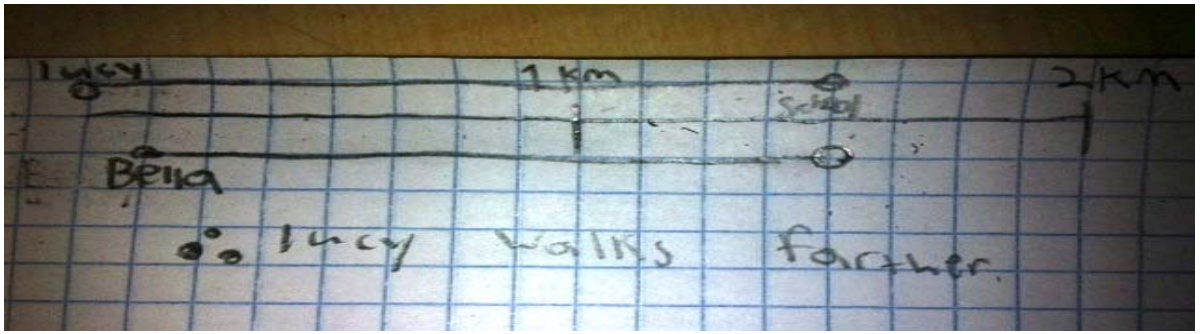
Fraction Situations

Lucy walks 1 1/2 km to school. Bella walks 1 3/8 km to school. Who walks farther? What picture would help represent this fraction story?





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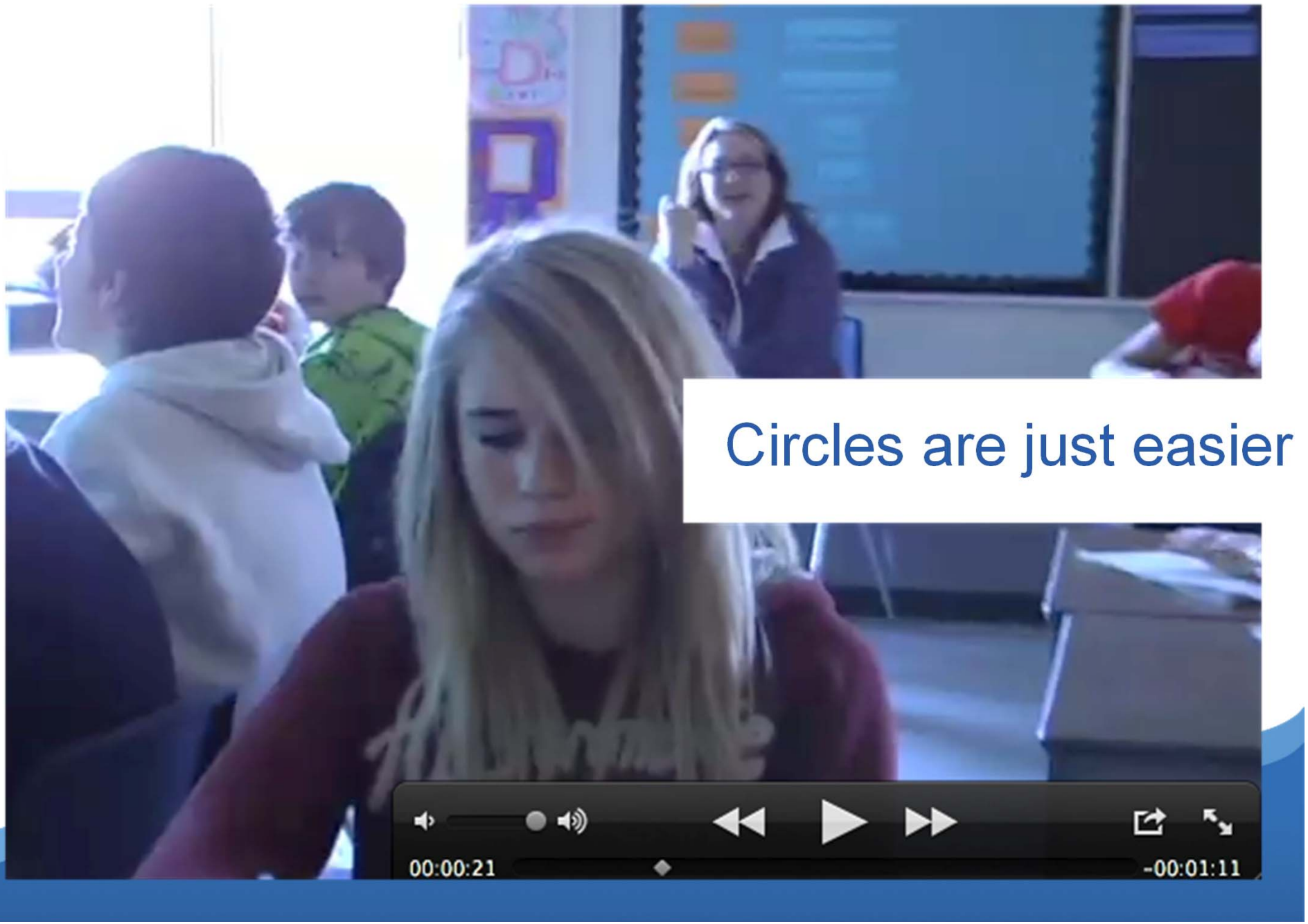
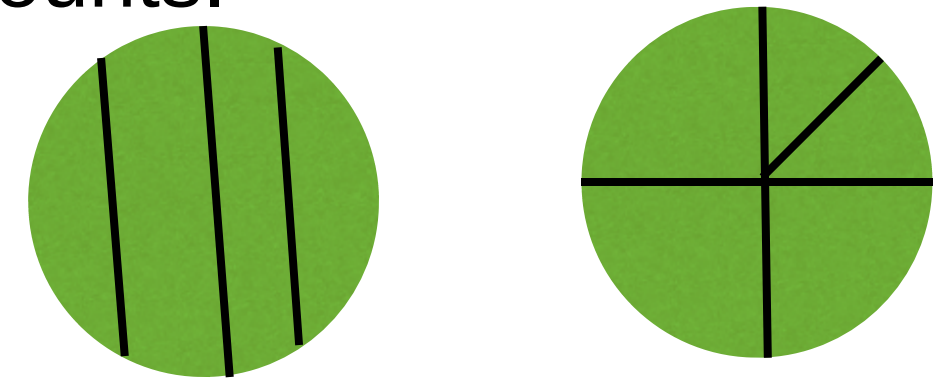

Circles are just easier
00:00:21
-00:01:11

But it simply isn't true…

1. They are hard to partition equally (other than halves and quarters)
2. They don't fit all situations
3. It can be hard to compare fractional amounts.



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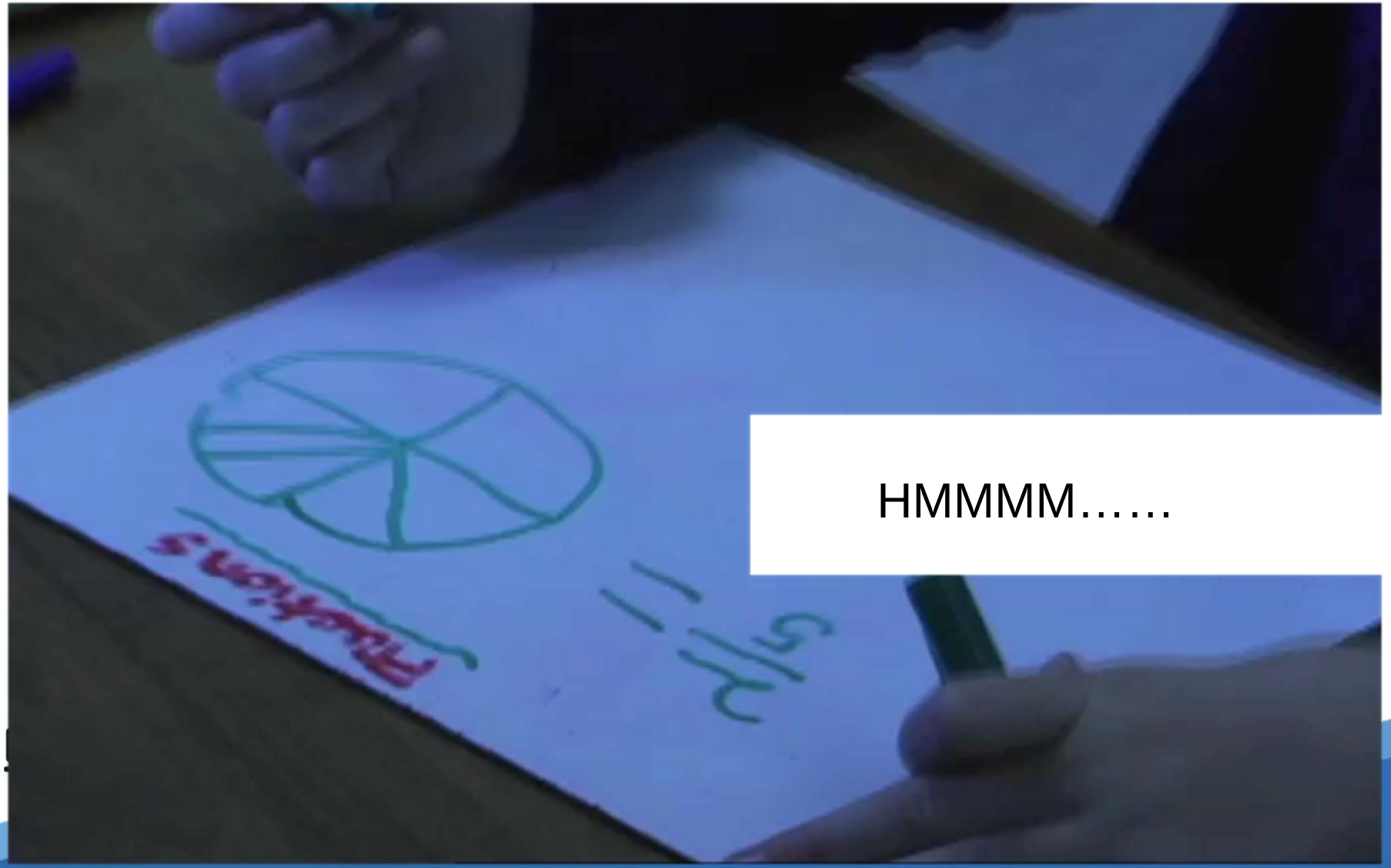
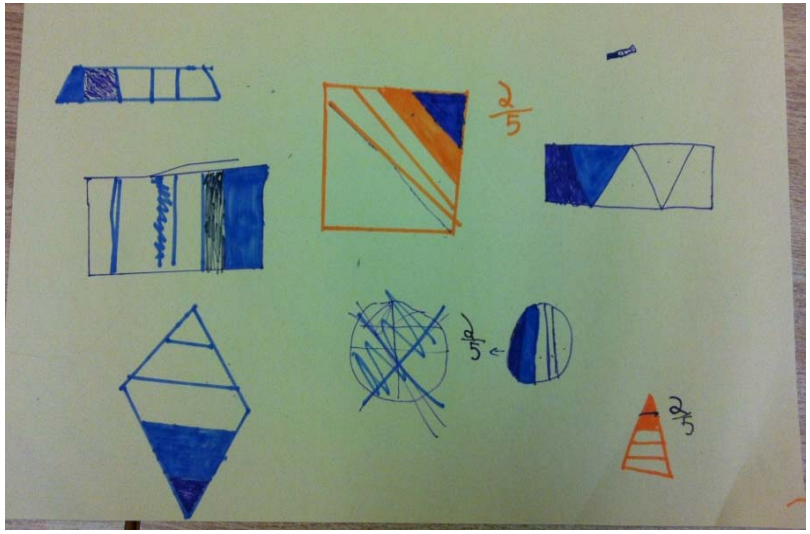
Students attempting to partition…



Over-reliance on circles to compare fractions can lead to errors and misconceptions…



No matter what the situation, students defaulted to pizzas or pies…

We had to teach another method for comparing fractions to move them forward…

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Number Lines

So we looked closely at linear models…

How do students:

-think about numbers between 0 and 1

-partition using the number line

-understand equivalent fractions and how to place them on the number line

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Why Number Lines?

Lewis (p.43) states that placing fractions on a number line is crucial to student understanding. It allows them to:

- PROPORTIONAL REASONING: Further develop their understanding of fraction size
- DENSITY: See that the interval between two fractions can be further partitioned
- EQUIVALENCY: See that the same point on the number line represents an infinite number of equivalent fractions

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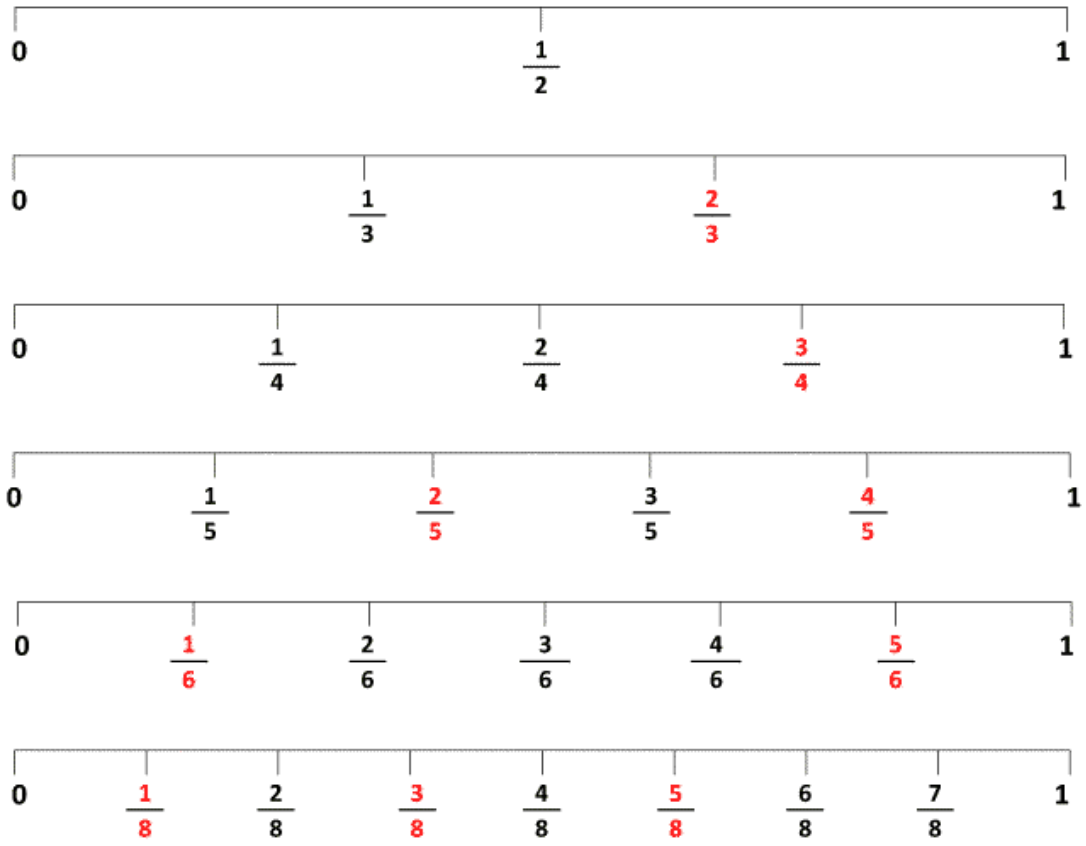
Fractions on Stacked Number Lines



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“Number line 0-4”

$\frac{1}{2}$

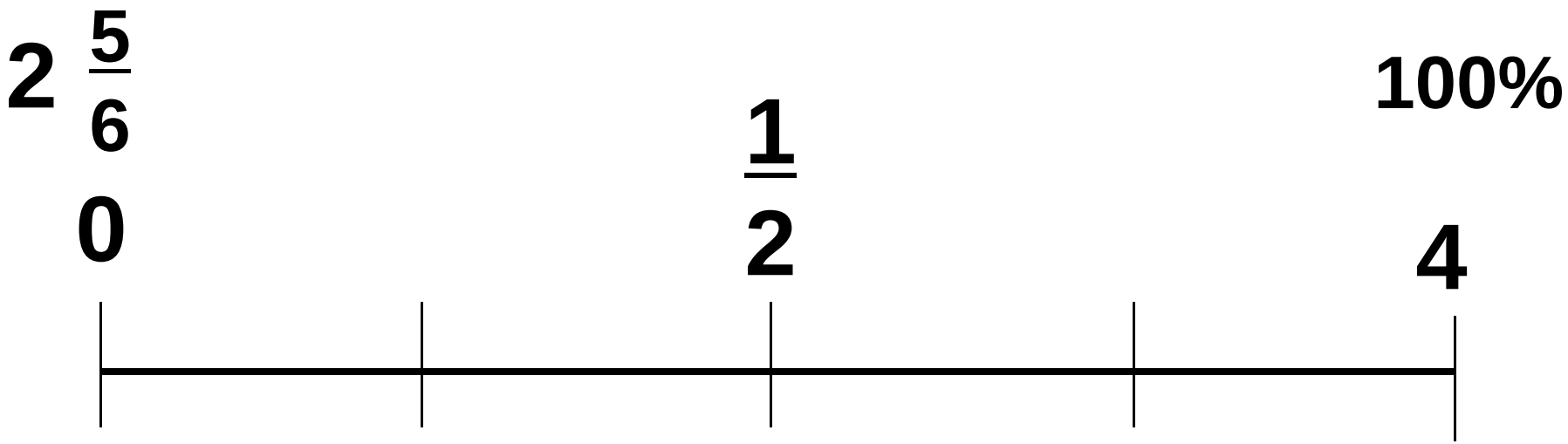
100%

$2\ \frac{5}{6}$

$\frac{7}{18}$

0.99

ORDERING THE FRACTIONS



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Implications for Teaching

- Connections: Have students compose and decompose fractions with and without concrete materials.
- Context: Get students to make better decisions about which representation(s) to use when.
- Exposure: Lots of exposure to representations other than part-whole relationships (discrete relationship models are important as well as continuous relationship models).

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Implications for Teaching

- Discussion/class math-talk to enhance the language of fractions, but also reveal misconceptions
- Use visual representations as the site for the problem solving (increased flexibility)
- Think more about how to teach equivalent fractions
- Think more about the use of the number line

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FRACTIONS Digital Paper

Professional Learning

Introduction

Professional Learning

- Implementing diagnostic tasks
- Co-planning, implementing and observing lessons
- Analysing student responses
- Sustaining focus on fractions

Fractions(content)

Instructional Decisions

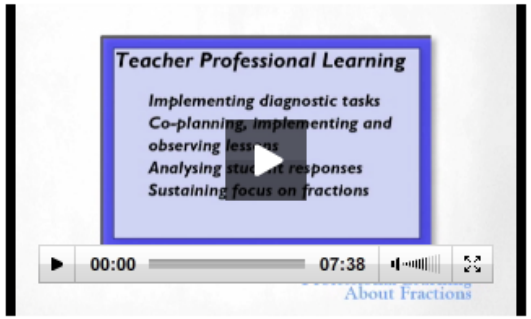
Student Learning

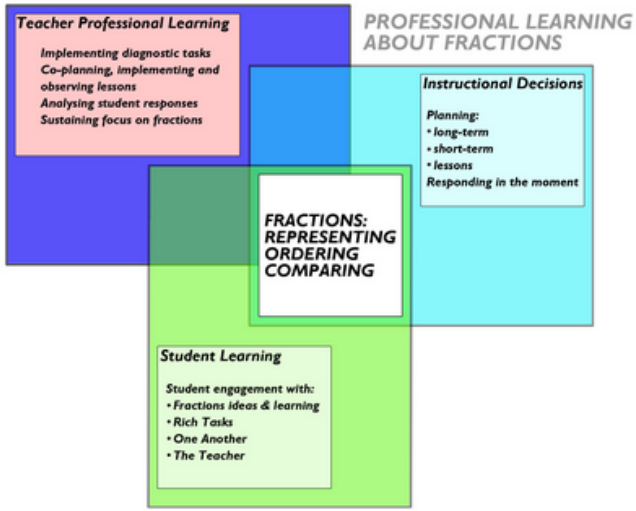
Resources:

Research Story

Professional Learning: Collaborative Action Research In each of the three school boards, teacher-researcher teams engaged in a process of collaborative action research (www.tmerc.ca/digitalpapers/) to explore the teaching and learning of fractions. Collaborative action research is a dynamic form of professional learning that engages educators and researchers in learning together by investigating areas of mutual interest. This occurred over the course of three to five release days (four to seven sessions) and involved a blend of in-class and out-of-class learning over a four-month period.

During the initial session, the teams explored different relationships represented and different actions implied by a fraction (see Math for Teaching Fractions resource). Teams also identified questions and dilemmas for further exploration. An interesting framework for thinking about these dilemmas is provided by the four categories below (Windschitl, 2002). These examples are drawn from the teams in this fractions action research





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